iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

A note on signals in electromagnetic barrier penetration

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1975 J. Phys. A: Math. Gen. 8 L113

(http://iopscience.iop.org/0305-4470/8/10/006)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.87
The article was downloaded on 02/06/2010 at 05:00

Please note that terms and conditions apply.

J. Phys. A: Math. Gen., Vol. 8, No. 10, 1975. Printed in Great Britain. © 1975

LETTER TO THE EDITOR

A note on signals in electromagnetic barrier penetration

A Kodre and J Strnad

Department of Physics and J Stefan Institute, University of Ljubljana, Ljubljana, Yugoslavia

Received 26 June 1975

Abstract. A previous calculation of delay time in electromagnetic barrier penetration is improved using a simple device for elimination of non-causal effects.

In a recent paper (Kodre and Strnad 1975) we investigated the transmission of electromagnetic signals across a barrier represented by a thin layer with index of refraction n_1 embedded in an optically denser medium with index of refraction n. At supercritical incidence, ie for angles of incidence θ greater than the critical angle of total reflection $\theta_c = \sin^{-1}(n_1/n)$, the effective group velocity inside the barrier exceeds the velocity of light *in vacuo* (Strnad and Kodre 1975). To examine these phenomena further we studied the wavefront velocity of a monochromatic signal with a sharp front:

$$A(t') = \mathrm{i}(2\pi)^{-1}\int_{-\infty}^{\infty}(p-\omega)^{-1}\exp(-\mathrm{i}pt')\,\mathrm{d}p\begin{cases}=0 & t'<0\\ =\exp(-\mathrm{i}\omega t') & t'>0\end{cases} \tag{1}$$

with $t' = t-(n/c)(x\sin\theta+z\cos\theta)$.

Applying the transmission coefficient $T(p,\theta)$ to each constituent plane wave of the signal, the transmitted part of the signal was obtained as

$$A_{\mathrm{T}}(t') = \mathrm{i}(2\pi)^{-1}\int_{-\infty}^{\infty}(p-\omega)^{-1}[\sin\delta/\sin(\delta-\mathrm{i}\alpha p)]\exp[-\mathrm{i}p(t'+t_Z)]\,\mathrm{d}p \tag{2}$$

with $t_Z = nZ\cos\theta/c$. At supercritical incidence δ and α are real constants, $\delta = 2\tan^{-1}(nn_1\epsilon/a^2\cos\theta)$ and $\alpha = n_1\epsilon Z/c$ where $\epsilon = (n^2\sin^2\theta/n_1^2-1)^{1/2}$ ($a = n$ for transverse electric (TE) and $a = n_1$ for transverse magnetic (TM) polarization) and Z is the layer thickness.

Integration in equation (2) can be performed by closing the integration path in the complex p plane. The result can be written in the form of a convolution

$$\begin{aligned}A_{\mathrm{T}}(t') &= (2\pi)^{-1/2}A(t')*\mathscr{T}(t',\theta)\\ &= \alpha^{-1}\sin\delta\exp[-\mathrm{i}\omega(t'+t_Z)]\int_{-\infty}^{t'+t_Z}\exp(\mathrm{i}\omega s-\delta s/\alpha)[1+\exp(-\pi s/\alpha)]^{-1}\,\mathrm{d}s\end{aligned} \tag{3}$$

of the incident signal (1) with the inverse transform of the transmission coefficient

$$\mathscr{T}(t,\theta) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}T(p,\theta)\exp(-\mathrm{i}pt)\,\mathrm{d}p. \tag{4}$$

The transform (4) represents the time dependence of the transmitted part of a short δ-function-like incident pulse. In other words, with the above transform the transmission coefficient $T(p, \theta)$ in the plane wave expansion of the signal is converted into a time-dependent Green function $\mathscr{T}(t, \theta)$.

There is an alternative interpretation of equation (3): an infinite monochromatic plane wave $\exp(-\mathrm{i}\omega t')$, $-\infty < t' < \infty$, is incident on the barrier and the transmission coefficient is switched on from zero to $T(p, \theta)$ at $t'(z = 0) = 0$, ie at the instant $t = nx \sin \theta/c$. (The layer is covered with a thin black sheet at the boundary plane $z = 0$ which is being gradually removed with constant velocity $c/n \sin \theta$ along the x axis.) Apart from the reflected signal, which we are not discussing here, both pictures are equivalent.

As already pointed out, the transmitted signal (2) at supercritical incidence has a quasistatic 'precursor'. This is a consequence of the simplicity of the model. The front of the signal penetrates the barrier at all times $t > -\infty$, and the perturbation within the barrier from earlier times always precedes the advent of the front to the barrier. This unsatisfactory feature of the model could be remedied by introduction of a side-truncated wave, which leads to a great mathematical complexity.

Another remedy of a more simple kind can be developed from the picture of the time-dependent Green function in the following way: let a plane wave with a sharp front (1) be incident on the barrier and the transmission coefficient be switched on from zero to $T(p, \theta)$ all along the barrier at the same instant $t = 0$. (The absorbing sheet at the boundary of the layer is removed instantaneously at $t = 0$.) By this device the side truncation of the incident signal is simulated. The resulting transmitted signal is obtained as

$$B_{\mathrm{T}}(t') = \alpha^{-1} \sin \delta \exp[-\mathrm{i}\omega(t' + t_Z)] \int_{t_Z}^{t' + t_Z} \exp(\mathrm{i}\omega s - s\delta/\alpha)[1 + \exp(-\pi s/\sigma)]^{-1} \, \mathrm{d}s. \qquad (5)$$

The approximate character of this calculational device does not allow us to observe any diffraction effects that occur at the newly created edge of the wave. Yet the non-physical 'precursor' of the earlier model has been evaded.

B_{T} vanishes when the upper integration limit falls below the lower limit, ie when $t' < 0$. Consequently, the wavefront progresses across the barrier with the same velocity c/n as in the surrounding medium and the transmission is thus shown to be subluminal for any angle of incidence. However, the form of the signal is considerably changed, the sharp front being smoothed into a linearly rising envelope.†

The expansion of the signal B_{T} into pole residua of integral (5) shows that contributions of all poles from the upper half-plane of p have vanished as a consequence of the absence of the transmitted field before $t' = 0$. This fact conforms with Fox's idea (Fox *et al* 1970) that the absence of singularities in the complex p plane above the path of integration is a necessary condition for signals to be causal. Though the idea has originally been developed for signal transmission in dispersive media it seems to be valid in connection with boundary condition dispersion too.

References

Fox R, Kuper C G and Lipson S G 1970 *Proc. R. Soc.* A **316** 515–24

Kodre A and Strnad J 1975 *J. Phys.* A: *Math. Gen.* **8** 533–8

Strnad J and Kodre A 1975 *Fizika* 7 13–21

† It is interesting to note that for critical incidence, $\theta = \theta_c$, $(B_{\mathrm{T}}{}^*B_{\mathrm{T}})_{\theta=\theta_c}$ has the same form as $(A_{\mathrm{T}}{}^*A_{\mathrm{T}})_{\theta=\theta_c}$ (see figure 1 in Kodre and Strnad 1975) if argument $t'' = t' + t_Z$ is replaced by t'.